

MTH 516/616: Topology II

Homework IV

(Due 14/04)

1. Show that the maps $G \xrightarrow{n} G$ and $H \xrightarrow{n} H$ multiplying each element by an integer n induce multiplication by n in $\text{Ext}(H, G)$.
2. Compute the simplicial cohomology groups of $S^1 \times S^1$, the Klein Bottle, and $\mathbb{R}P^2$ with respect to \mathbb{Z} and \mathbb{Z}_2 coefficients.
3. Show that if (X, A) is a good pair, where A is a closed subspace of X , then

$$H^n(X, A; G) \cong H^n(X/A; G),$$

for all n .

4. If A is a retract of X , then show that

$$H^n(X; G) \cong H^n(A; G) \oplus H^n(X, A; G).$$

5. Compute the groups $H^i(S^n; G)$ using Mayer-Vietoris sequence and cellular cohomology.
6. Show that if $f : S^n \rightarrow S^n$ has degree d , then $f^* : H^n(S^n; G) \rightarrow H^n(S^n; G)$ is multiplication by d .